



AN UNDERSTANDING OF BRAIN DISSIPATION USING HOPF ALGEBRA



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Abstract

In quantum field theory, we consider systems that are open and are in constant communication with a bigger system (a coupling to the bigger system) that is known as the thermal bath. The openness of the system enables this communication, albeit interaction. This communication or interaction is captured in one word-dissipation. In seeking to articulate the dynamics of the system's dissipation, we compute the details of the dynamics responsible for such dissipation, that is to say the Hamiltonian describing the system, the thermal bath and the system-thermal bath coupling. This is for the sake of attaining to a formalism of the system. We know that it is difficult, albeit impossible to compute a system that is open, this is because the exchange of energy, and fluctuation of forces between the systems, provides for an erroneous reading. Therefore in order to capture this

dynamics, the dissipation processes are enabled between the system and the bath, in such case, the system is generically closed. In such wise, we preserve the canonical structure of the canonical commutation relations (CCR). The issue we encounter in that which pertains to brain system, is in providing a formalism that will match the dynamics expressed in the usage of quantum field theory (QFT). We posit that an understanding of the brain system, can be better represented by borrowing excerpts from the formalisms as furnished by Hopf Algebra as used in Category Theory. We start off, by indicating that the dissipation is an integral aspect of human brain systems, and then we show that QFT is better adapted to providing an explanation to the aforementioned system. We then bring in the formalism provided by Hopf Algebra for a fitting conclusion to this.

Keywords: dissipation, brain, category theory, hopf algebra, biological system.

Introduction: Dissipation In Biological Living Systems.

The human brain is a living biological system, this is found in the dynamics that arises from it, that encapsulates both the conscious and the unconscious. As a biological organism (system), the structural composition of dissipation and its eventual evolution in time, is contained in a matter and energy communication. Owing to the fact that the brain

organism, is an open system, this communication is random, therefore leading to an increase of complexity which is found in the system. Again that it is an open system, its processes are not progressive or successive but spontaneous. E.B.P. Tiezzi et al., notes that

Interactions between system constituents, that are neither isolated nor free (that is isolated nor closed, addition mine), involve continuous reciprocal adaptation; each individual acts and reacts according to actions and reactions of the other individuals. This process, generated by competition and cooperation among the components, each pursuing its own aims, does not stop until organization that guarantees harmonious, non-conflicting interactions among individuals is achieved.¹

The dissipation as is being elaborated here is that which is made known according to QFT. It has been noted that dissipation is not provided for in Quantum Mechanics (QM), this is because in the QM, the doubling of the degrees of freedom is not allowed, and more to it, “the formalism of QM is based on conserving probabilities...”² Nonetheless, QFT is not QM, the differences are quite notable, however, we can summarize the differences noting that

¹ Elisa B.P. Tiezzi et al., “Dissipative structures in nature and human systems.” *Design and Nature* IV, 2008, 294-299.

² Vitiello, Giuseppe. “Quantum Dissipation and Coherence.” *HEP-TH-9503135*, 1995, 1-12.

in quantum mechanics, time is a parameter, while position is an observable. The preferential treatment of space and time violates relativity. In quantum field theories, position is no longer an observable; it is a parameter having the same status as time. The Lagrangian of closed systems do not depend explicitly on x .³

More to this, while we speak of first quantization to QM, we speak of second quantization to QFT. Nonetheless, second quantization does not mean that from classical theories, we quantize two times, No. the quantization is rather done once, but it is done in two steps namely

1. interpret these modes (the discrete modes) as having energy $E = \hbar\omega$ (this interpretation takes its initial stance from the classical theories)
2. quantize each mode as a harmonic oscillator.⁴

It would be again impossible to make use of classical physics to compute the dynamics of dissipation in biological system as the brain, since “the

³ Sunny Y. Auyang, *How is Quantum Field Theory Possible?* (Oxford: Oxford University Press, 1995), 51.

⁴ Matthew D. Schwartz, *Quantum Field Theory And The Standard Model*, (Cambridge: Cambridge University Press, 2014), 20

equations of motion defining the dissipative system requires the introduction of additional complementary equations”. Therefore, the required machinery used to acquire an understanding of dissipation in the brain, is (as proposed), QFT. However, QFT seeks to explain the fundamental properties in physics, using fields, at higher energies (which is synonymous with shorter scales. Like already noted, QFT does not have a mathematical formalism that is away or distinct from QM. This we noted when we insisted that in quantum physics, the quantization does not occur two times, but once.

Since dissipation works in an open system, and owing to the fact that for the sake of mathematical precision, the need to “close up the system”, is necessary. Therefore, in the means to understudy the open system, by means of “closing it up”, the only means to get past the reality of the damping and the fluctuations that arise do to the interaction, is to double the degrees of freedom (this is also to be referred to as the doubling of the freedom. In the move to double the degrees of freedom, the system can thus be closed for study, one that witnesses the system having a mirror image, which stands in as the thermal bath of the system. A fitting example to be used in the modelling of dissipative systems, is the damped harmonic oscillator(dho).

The classical equation of the dho is (on the x-axis)

$$m\ddot{x} + R\dot{x} + kx = 0 \quad (1)$$

where the parameters M , R , κ , are time independent {and since we have already asserted that the system in consideration is a dissipative one, a straight forward Lagrangian description that results in a consistent canonical formalism is near impossible. To remedy this, there would be the need to draw out a time reversed counterpart of equation (1), which is equation with respect to x , on the y -axis:

$$m\ddot{w} + R\dot{w} + kw = 0 \quad (2)$$

The equation (2) and (1), combine to form the Lagrangian of the entire system, as

$$L = M\dot{x}\dot{w} + 0.5R(\dot{x}w - x\dot{w}) - kxw \quad (3)$$

It should be noted that the Lagrangian for a closed system is

$$L = M\dot{\alpha}\dot{\epsilon} + 0.5R(\dot{\alpha}\epsilon - \dot{\epsilon}\alpha) - \kappa\alpha\epsilon \quad (4)$$

The equation (3) is the Lagrangian of the closed system that is the human brain, we ought to recall that the model used is seen in equation (4). Equation (2) stands as the reversed double of equation (1). This doubled image is what accounts for the quantum noise “in the fluctuating random force in the system-environment coupling”.⁶

⁶ Elenore Pessa and Giuseppe Vitiello, “Quantum Noise, Entanglement and Chaos in The Quantum Field Theory of Mind/Brain States.” *Mind and Matter* Vol.1, No.1, 2003, 1-15

The above formalism is drawn from the fact of the dissipation that occurs in an open system. In seeking to model a canonical formalism of the QFT of the brain, we have to make use of the damped harmonic oscillator (dho), and apply its findings to that of the brain. It is thus realized that since we cannot really model an open system, which always stands in interaction with other systems, and mainly its thermal bath, there comes the need to close the system, then analyse it as a closed system, even when it remains naturally an open system. The means to closing the system is by doubling the degrees of freedom. That is to say that when we double the degrees of freedom in the system, and that to which it is entangled to, we get a closed system that is ready to be studied. It has become evident that the quantum noise, the arrow of time (in what pertains to previous and present knowledge) and the doubling of the degrees of freedom are all indicative of the dissipative nature of the human brain system. This point is well summed up by Pessa and Vitiello, as they note that

Due to the permanent entanglement brain–environment, quantum noisy effects are intrinsically present in the brain dynamics. Moreover, we have seen that time evolution in the memory space may present chaotic behavior. It is interesting to remark that laboratory observations show an important role in brain dynamics of noise and of chaos. In the

dissipative model, noise and chaos turn out to be natural ingredients of the model. In particular the chaotic behavior of the trajectories in memory space may account of the high perceptive resolution in the recognition of the inputs. Indeed, small differences in the codes associated to external inputs may lead to diverging differences in the corresponding memory paths. On the other side, codes “almost” equal in all of their components may easily be recognized as being the “same” code (code identification, or “code–pattern” recognition).⁷

Quantum Dissipation

Even though quantum dissipation can occur in a plethora of quantum systems, our main focus in this paper is the brain. In the brain, we have already ascertained that dissipation occurs due to the coupling of the system, the brain, to its thermal bath. We can thereby assert that an entanglement occurs between both systems in the quantum realm that is visibly seen in the classical physics. We had earlier asserted in equation (4) that the Lagrangian of the closed system is a combination of the system and its double. The double of the system is that which the system is coupled or entangled to. Therefore for our interests as pertains to biological sciences and the brain, the double of the

⁷ Pessa and Vitiello, 1-15.

brain system is the environment, which is yet referred to as the thermal bath. Equation (4) arises from the intersection of equation (1) and (2), with the former connected to the x-axis and the latter to the y-axis. The Y-coordinate is the double of the system, and according to Vitiello, this is to account for the noise that is experienced in the quantum realm, that which he refers to as quantum noise. He notes that

When the classical solution $y=0$ is adopted, the x-system, the x-axis appears to be “incomplete”; the loss of information due to dissipation amounts to neglecting the bath and to the ignorance of the bath-system interaction, that is the ignorance of “where” and “how” energy flows out of the system. One can thus conclude that the loss of information occurring at the classical level due to the dissipation in the system, manifests itself in terms of “quantum” noise effects arising from the imaginary part of the action to which the y-axis' contribution is very important.⁸

The classical rendition of dissipation can be seen in metabolism, and this maintains the necessary homeostasis in the human brain-body entanglement. Quantum dissipation as spoken of in biological system, is subsumed under the auspices of thermo field dynamics (TFD).

⁸ Giuseppe Vitiello, “Links. Relating Different Physical systems through the common QFT algebraic structure.” arxiv:hep-th/0610094v1, 2006, 1-34.

TFD consists of computing a system further by enlarging the space of its framework to that of QFT, thereby allowing an ease of calculation, “with known techniques as the Feynman Rules and Diagrams, thermal averages of quantities by a simple pure state “vacuum” expectation value of a local operator”.⁹ TFD was developed by Matsumoto et al., in 1983 as a field theory for finite temperature, providing clear cut answers to calculation issues pertaining dynamical quantities.¹⁰ TFD considers systems whose temperature operate on a non-zero capacity. That is to say that such systems does not fall lower than zero. The dissipation that occurs in TFD, in the brain, requires the doubling of the degrees of freedom. This arises because while the human brain operates at a non-zero temperature, the environment in which it is coupled to, can operate at temperatures that greatly vary. Thus, to ensure that there is a control of the dynamics of the brain and its thermal bath, we double the degrees of freedom. According to Basti, Vitiello and Capolupo,

In TFD it appears clearly that the role played by the DDF is to include in the description the thermal bath, or environment, in which the system is embedded. In this way the global system (the system under study and its environment is a closed system and the canonical formalism of QFT can be used.¹¹

⁹ Tiago Santos. “Thermofield Dynamics.” Edisciplinas, 2019, 1-10. URL: https://edisciplinas.usp.br/pluginfile.php/5037795/mod_resource/content/1/Thermofield_Dynamics.pdf

¹⁰ Hiroshi Matsumoto, et al. “Thermo field dynamics in interaction representation.” *Prog. Theor. Physics* 70(2), 1983, 599-602.

¹¹ Gianfranco Basti, Antonio Capolupo and Giuseppe Vitiello, “The Doubling of the Degrees of Freedom in Quantum Dissipative Systems, and the Semantic Information Notion and Measure in Biosemiotics.” *Proceedings* 47(69), 2020, 1-7.

From the above equation, since (1) is coupled to (2), and we have already asserted that (2) is the reversed double of (1), when we vary (2) with respect to w (on the y -axis), working on (3), we get (1), and this is done when we vary (1), with respect to x , working on (3). Therefore (2) shows itself to be a time reversed copy or double ($w \rightarrow -w$) of (1). At such wise we define the momentum for the differing systems, p_x , p_w , as $p_x \equiv \delta L / \delta \dot{x} = m\dot{w} - 0.5Rw$; $p_w \equiv \delta L / \delta \dot{w} = m\dot{x} + 0.5Rx$. The Hamiltonian will thus be seen as

$$H = p_x \dot{x} + p_w \dot{w} - L = 1/m p_x p_w + 1/2m R(w p_w - x p_x) + (k - w^2/4m)xw \quad (5)$$

Since the quantum variables includes the energy and the momentum (x , p_x ; w , p_w), Equation (5) considers the total energy in the system, which includes the energy and the momentum. The canonical quantization for the above will include the introduction of commutators and the apparent annihilation and creation operators. The commutators $(x, p_x) = i\hbar = (w, p_w)$; $(x, w) = 0 = (p_x, p_w)$, with the apparent annihilation and creation operators as

$$a \equiv (\sqrt{0.5\hbar\Omega})[(p_x/m^{1/2}) - i(m)^{1/2}\Omega x] ; a^\dagger \equiv (\sqrt{0.5\hbar\Omega})[(p_x/m^{1/2}) + i(m)^{1/2}\Omega x];$$

$$b \equiv (\sqrt{0.5\hbar\Omega})[(p_w/m^{1/2}) - i(m)^{1/2}\Omega w] ; b^\dagger \equiv (\sqrt{0.5\hbar\Omega})[(p_w/m^{1/2}) + i(m)^{1/2}\Omega w]; \quad (6)$$

where the bosonic commutation relations reads is satisfied (which is $[A, B] \equiv AB - BA$), that is to say

$$[a\alpha, a^\dagger\beta] = \delta\alpha\beta \rightarrow [a\alpha, a\beta] = [a^\dagger\alpha, a^\dagger\beta] = 0 \quad (7)$$

From equation (7), we notice that there is a commutativity relations. However, there is no commutativity relations when the equation is as $[a\alpha, a^\dagger\alpha] = 1 = [a\beta, a^\dagger\beta]$.¹² These equations relate to the coupling that is seen between the system and the environment. These equations relate to the formalism that is inherent in the greater system (the system and its thermal bath). The “-” and “+” signs are interchangeable between the operator for the system and that of the thermal bath. Such that when these operators intersect, there is commutation relations and this is entirely indicative of dissipation in the brain.

Hopf Algebra as Bialgebra: A bialgebra is a vector space that is both an algebra and a coalgebra, for this to stand, the two algebraic systems must be compatible. This compatibility is defined by calling forth either one of the following equally linked conditions, which are algebra and coalgebra morphisms. Therefore we can say that a Bialgebra B, is composed of $(B, \Delta, \epsilon, \mu$ and $\eta)$, which is an algebra (B, Δ, η) , being augmented by ϵ and a coalgebra (B, μ, ϵ) , being augmented by η , making the diagram to be commutative:

¹² Vitiello Giuseppe, Enrico Celeghini, Mario G. Rasetti, “Quantum Dissipation. “Annals of Physics 215(1), 1992, 156-170.

$$\begin{array}{ccccc}
 & \Delta & & \mu & \\
 B \otimes B & \xrightarrow{\quad} & B \otimes B & \xrightarrow{\quad} & \\
 \downarrow \mu \otimes \mu & & \uparrow \varepsilon & \uparrow \eta & \uparrow \Delta \otimes \Delta \\
 B \otimes B \otimes B \otimes B & \xrightarrow{\quad} & k & \xrightarrow{\quad} & B \otimes B \otimes B \otimes B \\
 \text{id} \otimes \gamma \otimes \text{id} & & & &
 \end{array}$$

While Δ is the morphism of coalgebras, μ is the morphism for algebras, and $\gamma: B \otimes B \rightarrow B \otimes B$, is the linear map. If we take out the associativity of Δ and the coassociativity of μ , then B is said to be quasi bialgebraic. A bialgebra from the above diagram is thus B , which is equipped with a multiplication Δ and a comultiplication μ , a unit morphism η , and a counit morphism ε . Thus by its own definition, a bialgebra is self-dual, which indicates that if one can define a dual of B , then B is automatically a bialgebra. The commutative diagram above can be simplified thus:

$$\Delta: B \otimes B \rightarrow B, \eta: k \rightarrow B; \mu: B \rightarrow B \otimes B, \varepsilon: B \rightarrow k \quad (8)$$

Where Δ and η are coalgebra morphisms and μ and ε are algebra morphisms. It is quite necessary that we hint briefly on bialgebras because it is a stepping stone to arriving at the concept of a hopf algebra, which is actually a bialgebra with an antipode. And the concept of hopf algebra is important to the formalism of the dissipative quantum systems.

From the simplification of the commutative diagram, which is

$$\Delta: B \otimes B \rightarrow B, \eta: k \rightarrow B; \mu: B \rightarrow B \otimes B, \epsilon: B \rightarrow k,$$

we arrive at the concept of an antipode.

More on Hopf Algebra

It should be noted that if the antipode of a Hopf algebra is an anti-involution, such as

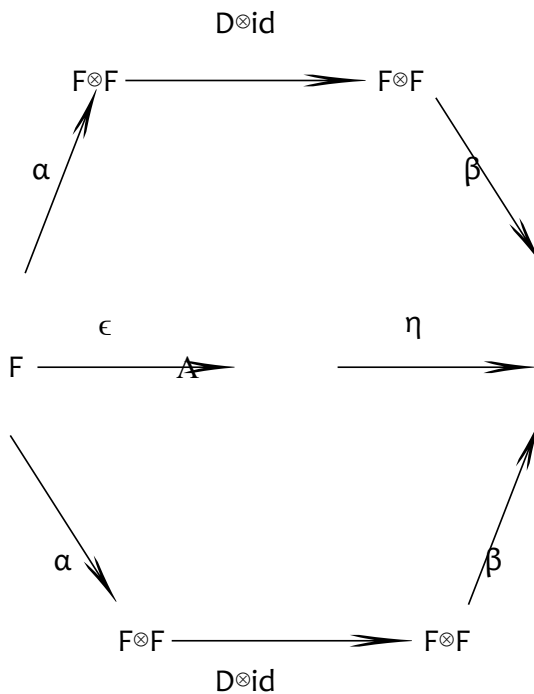
$$D \circ D = \text{id},$$

then we speak of an involutive Hopf Algebra.

That which is evident in the Hopf algebra is the fact of dual connections, that is to say products and coproducts, units and counits. This is reminiscent of Category theory, that has a plethora of dual connections. The duals are actually the reverse of the already mentioned connections, coproducts for products and colimits for limits. The coproduct is the dual of the product, while the colimit is the dual for the limit (other duals include pullbacks and pushouts, equalisers and coequalisers). What is being said here is that “by dualising limits we obtain colimits”,¹⁴ in the same way that budualising products we obtain coproducts etc.

¹⁴ Samson Abramsky and Nikos Tzevelekos, “Introduction to Categories and Categorical Logic.” arXiv:1102[math.CT], 7th Feb 2011, 1-101.

Since the Hopf Algebra is a bialgebra that takes into its stride units and counits, we can therefore represent it as such using the notation of F-bialgebra, with a linear function $D:F \rightarrow F$ (that is referred to as the antipode). therefore the diagram commutes:



Where α is the comultiplication and β is the multiplication, η is the counit and ϵ is the unit. Hopf Algebra is dual in its presentation. Duality reflects the correlation between the properties of the category C and that of its opposite C^{op} . When a proposition is

given, considering the category C , we only interchange the source and the target of each morphism and also the order of the composition of the two morphism, in order to get the opposite category C^{op} . Hopf Algebra attests to the fact that the two opposing category, that is to say the category and its opposite can stand side by side and then there is a commutation relation between the two categories. Duality as noted here is expansive in Category theory, where we have the likes of limits and colimits, pullbacks and pushouts etc. There is yet the duality between stone spaces and the Boolean algebra, there is also that between an end and a coend.¹⁵

The epistemological importance of the duality principle as highlighted so far in Hopf Algebra (Category Theory) is to emphasize the fact that there even amidst its many approaches, the truth remains invariant under any kind of transformation. We are thus saying that if a statement is true in C , then it is true in its dual C^{op} , the same applies if the statement is false in C . More to this, if in a category, the commutative relation as contained in association and unit, holds, then it should likewise hold in the dual category. By means of the duality principle, we are able to variedly alter the objects and the arrows, yet we maintain the foundational truth that inheres therein. Do duals communicate or interact? This has to be an arena for an

¹⁵ Given a functor $F: C^{op} * C \rightarrow X$, an end of F in X is a universal wedge $e: w \rightarrow F$, such that any other wedge $e': w' \rightarrow F$ factors through e via a unique map $w' \rightarrow w$. The notion of a coend is dual to the notion of an end

interestingly different research, yet suffice it to say that it is seemingly possible since, there is a line of connection between the duals, namely the truth that exists therein. Duality also inclines us towards the thought of “doubling”, this is because the dual is somewhat a double (lookalike) of that which it represents, the difference being in the swapping of the target and initial objects, as well as the composition and morphisms.

Hopf Algebra and Dissipation

The formalisms of a dissipative quantum system, as seen in thermal field dynamics and quantum field theory, incorporates the element of doubling as a means to giving credence to its definition. This is because, in the words of Arimitsu, Umezawa and Yamanaka,

the quantum field theory provides us with a useful formalism for quantum systems with infinite degrees of freedom. A program of reformulating the nonequilibrium quantum statistical mechanics with infinite degrees of freedom in the terminology of quantum field theory has been developed by extending thermo field dynamics (TFD) which is quantum field theory with thermal degrees of freedom. This TFD... is built on the concepts of thermal doublets... Thus with any operator A there is associated another operator $\tilde{A}$ which is called the conjugate of A . The doublet A_μ with

$A_1 = A$ and $A_2 = \tilde{A}$, is called a thermal doublet.¹⁶

Doubling of the degrees of freedom, partitions the means to arriving at an all-inclusive description of the formalisms of dissipative systems, this is because when a system is engaged in dissipation, it only means that it is coupled to a thermal reservoir or bath, as captured by Arimitsu, Umezawa and Yamanake. To better conceptualise this doubling of degrees of freedom, which accounts for the “infinitely many unitarily inequivalent representations of the canonical (anti-) commutation relations”,¹⁷ the concept of coalgebras with respect to “algebraic doubling” that is well witnessed in the self-dual nature of the bialgebras, but for our purposes, the q -deformed Hopf Algebra, which is a type of bialgebra, will be used.

The Hopf algebra as we have already seen is a type of bialgebra, in that it includes the duality of the algebra and coalgebra. As pertaining to the formalism of the QFT, the coalgebraic aspect of the Hopf algebra is singled out. In the coalgebra, the existence of the coproduct, which duplicates the product of the algebra, indicates already a doubling. According to Blasone et al,

¹⁶ Toshichico Arimitsu, Hiroomi Umezawa and Yoshiya Yamanaka, “Canonical Formalisms of Dissipative Fields in Thermo Field Dynamics.” *J. Math. Phys.*, 28(11), 1987, 2741-2752.

¹⁷ Vitiello Giuseppe, “Links. Relating Different Physical Systems through the Common QFT Algebraic Structure.” *arXiv:hep-th/0610094v1*, 9th October 2006, 1-23.

Coproducts are commonly used in the familiar addition of energy, momentum, angular momentum and of other so-called primitive operators. The coproduct of a generic operator O is a homomorphism defined as

$$\Delta O = O \otimes 1 + 1 \otimes O \quad O_1 + O_2, \text{ with } O \in A. \quad (9)$$

Since additivity of observables such as energy, momentum, angular momentum, etc. is an essential requirement, the coproduct, and therefore the Lie–Hopf algebra structure, appears to provide an essential algebraic tool in QM and in QFT.¹⁸

This doubling does not indicate an excess neither is it unnecessary, it is albeit quite important in denoting and making clear a full description of the observable variables in the system. This point is captured by Basti, Capolupo and Vitiello, who note that

...in order to specify, e.g., the total energy E of two identical particles, one writes $E = E_1 + E_2$. The meaning of the labels in such a formula is that E_1 refers to the first particle, and E_2 to the second particle. However, it is easy to realize that $E_1 = E \times 1$ where the index 1 thus refers to the first position. Similarly, $E_2 = 1 \times E$. Thus, $E = E_1 + E_2 = E \times 1 + 1$

¹⁸ Massimo Blasone, Giuseppe Vitiello and Petr Jizba, *Quantum Field Theory and its Macroscopic Manifestations. Boson Condensation, Ordered Patterns and Topological Defects*, (Covent Garden-London: Imperial College Press, 2011), 207.

$\times E$, and similarly $J = J_1 + J_2 = J \times 1 + 1 \times J$, for the angular momentum, which are nothing but the commutative coproducts of a coalgebra. Here, “commutative” refers to invariance of the coproduct under the permutation $1 \leftrightarrow 2$, as it needs to be on the premise that the particles are identical. We should be able therefore to go from the algebra A for the single particle to the algebra for two of them, namely, $A \rightarrow A \times A$. Of course, we need also to be able to go back to a single particle, namely, $A \times A \rightarrow A$.¹⁹

The coalgebra (coproduct) is used to negotiate elements in the system, as already indicated, as energy, momentum, angular momentum. And this same edifices used in energy, can be well applied to that of momentum and angular momentum. That is to say that if the energy E of two identical particles, one writes $E = E_1 + E_2$. The meaning of the labels in such a formula is that E_1 refers to the first particle, and E_2 to the second particle. We can thus write it as $E = E_1 + E_2 = E \times 1 + 1 \times E$, and similarly $J = J_1 + J_2 = J \times 1 + 1 \times J$, for the angular momentum, also similarly $P = P_1 + P_2 = P \times 1 + 1 \times P$. It ought to be noted that the coalgebra that is used here is the non-commutative. Non commutative does translate to the fact that there is no single morphism

¹⁹ Gianfranco Basti, Antonio Capolupo and Giuseppe Vitiello, “Quantum Field Theory and Coalgebraic Logic in Theoretical Computer Science”.

that defines the objects in the category or said in another way, it implies that at any point the order of operation is changed, it affects the resultant effect of the operation (this is in contrast to commutativity in vector parlance).

The coproduct makes it possible for operating on two modes, and an example to denote this is the addition of the angular momentum J_α , $\alpha = 1, 2, 3$, of two particles:

$$\Delta J_\alpha = J_\alpha \otimes 1 + 1 \otimes J_\alpha \equiv J_{\alpha 1} + J_{\alpha 2}, J_\alpha \in SU(2).^{20} \quad (10)$$

To make this coproduct understandable, according to Blasone et al., we make use of the bosonic algebra named $h(1)$

...generated by the set of operators $\{a, a^\dagger, H, N\}$ with commutation relations:

$$[a, a^\dagger] = 2H, [N, a] = -a, [N, a^\dagger] = a^\dagger, [H,] = 0. \quad (11)$$

Here a and $a^\dagger$ denote the generic annihilation and creation operators. H is a central operator, constant in each representation and “” denotes any of the $a, a^\dagger, H, N$. The Casimir operator is given by $C = 2NH - a^\dagger a$. The algebra $h(1)$ is a Hopf algebra,

²⁰ Giuseppe, Vitiello, “How to Deal With The Arrow of Time in Quantum Field Theory.”
 arXiv:hep-th/0110182v1, 19th October 2001, 1-16.

equipped with the coproduct operation, defined by

$$\Delta a = a \otimes 1 + 1 \otimes a \equiv a_1 + a_2, \quad (12)$$

$$\Delta a^\dagger = a^\dagger \otimes 1 + 1 \otimes a^\dagger \equiv a^\dagger_1 + a^\dagger_2, \quad (13)$$

$$\Delta H = H \otimes 1 + 1 \otimes H \equiv H_1 + H_2, \quad (14)$$

$$\Delta N = N \otimes 1 + 1 \otimes N \equiv N_1 + N_2. \quad (15)$$

Note that $[a_i, a_j] = [a_i, a^\dagger_j] = 0; i, j = 1, 2, i \neq j$.²¹

In the such wise, we can introduce the q -deformed Hopf algebra, in similar fashion as that of the bosonic algebras, such as

The q -deformation of $h(1)$ is the Hopf algebra $h_q(1)$:

$$[a_q, a^\dagger_q] = [2H]_q, [N, a_q] = -a_q, [N, a^\dagger_q] = a^\dagger_q, [H,] = 0 \quad (16)$$

where $N_q \equiv N$, $H_q \equiv H$, and we introduce the Casimir operator C_q is given by $C_q = N[2H]_q - a^\dagger_q a_q$. The deformed coproduct is defined by

$$\Delta a_q = a_q \otimes q^H + q^{-H} \otimes a_q, \quad (17)$$

$$\Delta a^\dagger_q = a^\dagger_q \otimes q^H + q^{-H} \otimes a^\dagger_q, \quad (18)$$

$$\Delta H = H \otimes 1 + 1 \otimes H, \quad (19)$$

$$\Delta N = N \otimes 1 + 1 \otimes N, \quad (20)$$

²¹ Massimo Blasone, Giuseppe Vitiello and Petr Jizba, Quantum Field Theory and its Macroscopic Manifestations. Boson Condensation, Ordered Patterns and Topological Defects, 207.

whose algebra is isomorphic with (18): $[\Delta a q, \Delta a^\dagger q] = [2\Delta H]q$.²²

We assert before all else that the categories of the Hopf bialgebra (coalgebra/algebra) and that of the Hilbert spaces are self-dual. With that being said, it is further asserted that this

...self-duality is instead non valid in the usage of both these structures in thermal QFT, where the thermal deformation parameter q breaks the symmetry coalgebra-algebra of Hopf bi-algebras, making non-commutative the co-products of the coalgebra, and, correspondingly, the composite Hilbert space includes the system states H_A and their entangled environment states $H_{A\bar{A}}$, according to the tensorial product $H_{A\bar{A}} \equiv H_A \otimes H_{\bar{A}}$.²³

For Hopf algebra, (and/or) Hilbert spaces, what we routinely consider is the fact that the system is engaged in a dissipative dynamic, and as Vitiello notes:

Dissipative systems from the point of view of the quantum theory for Brownian motion have been analyzed in the path integral

²² Blasone, ²⁰⁸

²³ Gianfranco Basti, Antonio Capolupo and Giuseppe Vitiello, "Quantum Field Theory and Coalgebraic Logic in Theoretical Computer Science".

formalism by Schwinger and by Feynman and Vernon and are of course a major topic in non-equilibrium statistical mechanics and non-equilibrium Quantum Field Theory (QFT) at finite temperature.²⁴

Therefore a fundamental theory (fundamental here used insists on the microscopic theory) for a dissipative system, will incorporate every detailed process that gives rise to dissipation, of which includes the quantum effects. We may proceed by calculating the total energy, that is the Hamiltonian of the system, the bath to which the system is entangled to and the interaction of the system and the bath.²⁵

As already succinctly stated, the problem with QM, as regards that which pertains to dissipative systems, arises from the fact of the unitarily equivalent representations of the CCR. This is further hinged on fact of the formalism of QM, which is based on the conservation of probabilities. At the instance of such, the time-irreversible evolution (which Vitiello calls the arrow of time), is not depicted by QM.²⁶ If we are to fall to classical mechanics for aid in explanation, we end up introducing more complementary equations, which yet will not be able to account for the necessary element of the time evolution. Yet in QFT it is, however, “possible

²⁴ Giuseppe Vitiello, “Quantum Dissipation And Coherence.”
arXiv:hep-th/950313v1, 21st March 1995

²⁵ Vitiello, “Quantum Dissipation And Coherence.”

to describe time-irreversible evolution by resorting to the existence of infinitely many unitarily inequivalent representations of the canonical commutation relations (ccr)".²⁷ In depicting the formalism of a dissipative system (one should always be reminded that the system we are considering is the brain system that is entangled to its thermal bath, the environment), which is an open system, we realize that the arrow of time plays an integral role in determining the placement of the present and past events. This is encountered in the fact of "getting an information", as this brings to bear a division in the time coordinates, such that one is able to differentiate between a before "getting an information" and an after "getting an information".²⁸

Conclusion: Brain Dissipation and Hopf Algebra Duality

The Hopf Algebra used here only attests to the dissipation of the brain, we did not engage in the inner dynamics of the Hopf Algebra system, save that which underscores brain dissipativity. This is the reason why it is used in line with the findings provided for by Category Theory. That is to say that the article makes use of Hopf

²⁶ Vitiello, "How To Deal With The Arrow Of Time." ¹⁻¹⁶

²⁷ Vitiello, "How To Deal With The Arrow Of Time." ¹⁻¹⁶

²⁸ Elenora Pessa and Giuseppe Vitiello, "Quantum Noise Induced Entanglement and Chaos in the Dissipative Quantum Model of the Brain." *International Journal of Modern Physics B*, Vol ¹⁸, No. ⁶, ²⁰⁰⁴, ¹⁻¹⁴.

Algebra as regards Category Theory.

There are philosophical issues that arise as one ponders on QBD, and they include the problem of consciousness and memory. There is also the application of the findings of QFT in the human brain that is noisy and wet, which implies that the brain is not suitable to fall under the algorithms provided for by QFT. To these issues, QBD, robustly answers. Nonetheless, they are not included in the work because that is not the precise need upon which the article is based. However, I have treated these issues in two papers already written titled, ***Apparent Limitations To The Orchestrated Objective Reduction Of Penrose And Hameroff As Regards Consciousness***, which treats the issue of the brain as a noisy and wet environment in respect to QFT and ***Quantum Brain Dynamics. A Possibility of Having a Quantum Interpretation of the Brain*** which considers the issues of consciousness and memory.

Dissipation as we have already noted envelopes a system in interaction with another, which stands as its thermal bath. Again we have seen that in order to denote such dynamics we have to double the degrees of freedom. It is in doubling this degrees of freedom, that we make use of the hopf algebra, or better put the q-deformed hopf algebra (quantum deformed hopf algebra). It ought to be reiterated that doubling the degrees of freedom, enables an all-inclusive description

of the formalism of the systems in dissipation. The duality in the overall system (comprising of the system and the environment), is actually the two systems coupled to each other. That is to say that between the system and the environment to which the system is entangled to, there is a inverse representation, a mirror-like representation. This is what G. Vitiello calls the double, as he notes saying that “In the dissipative quantum model the intrinsic dissipative character of the brain dynamics strongly points to consciousness as dialogue with the inseparable own Double”.²⁹ The duality of the dissipative system, leads us back to the formalism as espoused by the dho (in eqs. 1-7). It might be asked that if this formalism as provided in eqs. 1-7, is that of the system and its thermal bath, then why bring in that provided by Hopf Algebra? The answer is that, owing to the duality that exists between the system and its bath, which occasions the doubling of the degrees of freedom, and which proscribes that the thermal bath is nothing more than the inverse double of the system, its inverse dual, as we see in Hopf algebra. We therefore arrive at the importance of Hopf Algebra to brain dissipation.

Brain dissipation comes on the heels of the spontaneous breaking of symmetry (symmetry is broken spontaneously when the ground state

²⁹ Giuseppe Vitiello “Quantum Dissipation And Information: A Route To Consciousness Modeling.” *NeuroQuantology* 1(2), 2003, 1-16.

equations is not the same as the dynamical equations).³⁰

We should note that in brain dissipation using QFT, brain states are considered to be of finite temperature (this is biologically confirmed), and there is a time evolution that is irreversible (which is always pointing forward, that is linearly). The finitude of the temperature and the irreversible time evolution in QFT require the doubling of the degrees of freedom such as $A_\kappa : A_\kappa \rightarrow A_\kappa \times \tilde{A}_\kappa$, for any κ . In which, $\tilde{A}_\kappa$ represents the environment, to which the system is embedded. From the aforesaid, we see how the comultiplication function as seen in hopf algebra is introduced. Therefore the doubling operation finds its formalism contained in that of Hopf Algebra. This doubling operation describes the interaction between the system and its thermal bath, leading us to consider the overall system, that is the $\{A, \tilde{A}\}$ system as a closed system. Thus, it allows the use of the QFT canonical formalism, which is indeed limited to closed systems.

³⁰ Once the external stimuli is intercepted by the body, a necessary SBS occurs, a definite ground state, from the many infinite ground states or vacua is chosen. Massless NGB are observed, which carry long-range correlation waves observed in the brain dynamics. As a consequence of this, the time-reversal symmetry is also broken, because the choice of a ground state, entails that the information has been recorded. After information has been recorded, the brain state is fixated and the brain cannot be brought to the state configuration in which it was before the information printing occurred, for before the information recording process, the brain can in principle be in anyone of the

To fully embellish this summarily we have to note that:

1. In a dissipative (non-equilibrium) quantum system, there are (in principle) infinitely many quantum vacuum's (ground or zero-energy) states, on each of which a whole set of non-zero energy states (or “state space” or “representation states”) can be built.³¹
2. Each input triggers one possible irreversible time-evolution of the system, by inducing a “symmetry breakdown” in one quantum vacuum, i.e., by inducing in it an ordered state, a coherent behavior, effectively “freezing” some possible degrees of freedom of the constituting elements behaviors (e.g., by “constraining” them to oscillate on a given frequency), in the same time “labeling” it as the coherent state induced by that input, as an “unitary non-equivalent state” of the system dynamics. In fact, such a coherent state persists in time as a ground state (DWQ are not energetic bosons, are Goldstone bosons)... A brain no longer coupled with its environment is either in a pathological state, or it is directly dead.³² And
3. At this point emerges the DDF principle as a both physical and mathematical necessity of the model.

³¹ infinitely many (unitarily inequivalent) vacua
Gianfranco Basti. “The Notion Of Information In Quantum Physics And
The Dual Ontology Of Nature.” STOQATPUL, 1-62, URL:
<http://www.stoqatpul.org/lat/materials/basti_qft_dual_ontology.pdf>
in (13/03/2021).

³² Basti, 1-62,

Physical, because a dissipative system, even though in non-equilibrium, must anyway satisfy the energy balance. Mathematical, because the $o(\text{zero})$ energy balance requires a “doubling of the system degrees of freedom”. The doubled degrees of freedom, say $\tilde{A}$ (the tilde quanta, where the non-tilde quanta A denote the brain degrees of freedom), thus represent the environment to which the brain is coupled. The environment (state) is thus represented as the “time-reversed double” of the brain (state) on which it is impinging. The environment is thus “modeled on the brain”, according to the finite set of degrees of freedom the environment itself elicited. Anyway, which are the available degrees of freedom to be elicited for that input depends on the brain itself that, for this reason, is effectively a self-organizing system.³³

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³³Basti, 1-62,

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