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MODELING AND OPTIMIZATION OF CONCRETE MIX PROPORTIONS FOR ENHANCED SLUMP AND COMPRESSIVE STRENGTH

Ezechukwu Obiora Patrick

Department of Civil Engineering Technology, School of Engineering Technology
Federal Polytechnics Oko

obioraezechuxdaddy8@gmail.com

&

Onuamah Patrick, PhD, & Anajiofor-Ike Sandra Chinenye, PhD

Department of Civil Engineering, Enugu State University of Science and Technology (ESUT),
Enugu

patofinda@gmail.com & chinenye.ike@esut.edu.ng

Abstract

Traditionally, concrete mixture proportioning (mix design), usually based on a trial-and-error process, cannot objectively determine the best component mix to meet multiple performance criteria simultaneously. Furthermore, the method of trial mixes does not provide a means to obtain prediction equations for the various performance criteria which can be subsequently used to obtain optimal combinations of the concrete constituent mix proportion. A statistical constituent mix design approach provides a useful and cost-effective means of concrete performance optimization. In this work, using four mixture components, namely coarse and fine aggregates, cement and water, the slump and compressive strength of mix proportions were optimized by employing a statistical mixture approach based on IV-optimal and ANOVA design. The key performance criterion used was compressive strength: compressive strength of range $20 - 30\text{N/mm}^2$. Test results from 20 experimental runs were investigated to develop models. The results from verification tests confirmed the expected predictions by the fitted models.

Keywords: Modeling, Optimization, Concrete Mix Proportions, slump, compressive strength

Introduction

Concrete is obviously one of the most widely used construction materials due to its properties, such as versatility, durability, and strength. The performance of concrete, particularly its slump (workability) and compressive strength, is influenced by various mix proportions and environmental factors. Achieving the desired workability (slump) and strength (compressive strength) requires precise mix proportions, as emphasized by Mehta and Monteiro (2014), optimizing these properties is crucial for both structural integrity and construction efficiency.

Concrete mix proportioning methods and guidelines- ACI 211.1(R2009) and ACI 363-1997- provided the techniques for proportioning a given mixture, though they have not provided steps for obtaining the optimal mixtures with the best performance at the lowest cost. In the concrete methods and guidelines Trials and Error method is used to arrive at the desired mix proportion for a certain compressive strength. This is usually carried out by varying an independent variable at a time. However, this approach never objectively provides the best proportioning of constituents to obtain the desired compressive strength, as it does not allow investigating interactions among the independent variables and also does not develop prediction equations for modeling and optimization. On the other hand, the use of statistical principles in developing concrete mix proportions will improve the results in the concrete production stage and hence allow useful data to be obtained without so many combinations of component proportions (Lawler et al., 2005). In using the statistical method, a series of procedures ought to be taken to obtain the most optimal result.

The modeling and optimization of slump and compressive strength in concrete mix proportions are critical for enhancing the performance and sustainability of concrete structures. Lasisi and Osunade (1985) observed that an increase in sand/cement content and decrease in water/cement ratio resulted in increased compressive strength of concrete. Lasisi and Ogunjide (1984) established that the higher the laterized/cement ratio, the lower the compressive strength, and the fewer the grain size range, the higher the compressive strength. I-Cheng (1999) and Yeh (2009) described methods for predicting slump and compressive strength of high-performance concrete.

In this work, using four mixture components as follows: coarse and fine aggregates, cement and water, the slump and compressive strength of mix proportions were optimized by employing a statistical mixture approach based on IV-optimal and ANOVA design. The key performance criterion used was compressive strength: compressive strength of range 20-30N/mm² was used. Test results from 20 experimental runs were investigated to develop models. The results from verification tests confirmed the expected prediction by the fitted models.

Statement of the problem

In the past, achieving a specific compressive strength required tedious trial and error with various mix proportions, which was both wasteful and inefficient. Even after the effort, the desired strength was often not reached, making the process futile. Concrete performance depends largely on the proportioning of its constituent materials, with slump and compressive strength being the primary indicators of workability and structural performance. However, achieving an optimal balance between these properties remains a major challenge because improvements in workability may adversely affect strength, while conventional concrete mix design methods rely on repeated laboratory trial mixes that are time-consuming, labour-intensive, costly, and often unable to identify the most efficient mix proportions. Although advances in statistical and computational techniques have made it possible to model and optimize concrete mix designs more accurately, their application in concrete production remains limited, particularly in developing countries. Furthermore, the complex interactions among mix constituents make it difficult to determine optimal proportions using

traditional methods alone. Existing studies have largely focused on either predicting compressive strength or optimizing concrete mixtures separately, with relatively few integrating both modeling and optimization to simultaneously enhance slump and compressive strength. Therefore, there is a need to develop a comprehensive modeling and optimization framework capable of accurately predicting concrete performance and identifying optimal mix proportions that improve workability and strength while minimizing material consumption, production costs, and experimental effort, thereby promoting more efficient and sustainable concrete production.

Objective of the Study

The study set out to:

1. Develop regression models predicting the properties of interest as well as investigating the effect of four components and their blending effects to reach an optimal mix proportion using a commercially available software.

Significance of the Study

Below are the categories of people that will benefit from the outcome of the study. They include:

1. Civil engineers and construction professionals

The knowledge got from this study will improve concrete mix designs reduction of wasteful time and effort in carrying out trial mixes.

2. Regulatory bodies

The study will be of great help to the regulatory bodies in setting industry standards on concrete quality.

3. Environmentalists

Environmentalists will benefit from the study effectively because the results of the study will promote a reduction in material waste and sustainability in the construction industry, thereby encouraging a green environment.

4. Construction companies

The construction companies will enjoy cost-effectiveness and efficient building practices.

5. Project developers

The outcome of the study will help the project developers in optimizing construction time and material use.

6. Researchers and academics

The study will serve as a reference for future studies in concrete technology.

Materials and Methods

Compressive Strength Test: Concrete produced from the Aggregates

The main objective of this test is to obtain the strength of a concrete mix produced from these aggregates sourced from different localities. The strength of the aggregate used in the production of concrete usually plays a significant role in the compressive strength of the concrete produced, and concrete constitutes up to 75% of the common materials used in construction works; it will be of interest to verify the effect of these aggregates from different sources by using them in casting concrete cubes.

Apparatus

1. Metal mold
2. Tapping rod
3. Tray
4. Compression machine

5. Knife edge
6. Trowel.

Preparation of the samples

The concrete constituents were mixed using a prescribed mix of 1:2:4. The mold used is cubed to accommodate a cube specimen of 15 x 15 x 15cm. The joints between the sections of the mold are coated with oil or grease for easy tightening and loosening of the nuts.

Procedure

The sample (cement, sand, coarse aggregate) was put in a tray and mixed thoroughly with a trowel using the required water-cement ratio. After a thorough mix, the fresh concrete was placed in a cube mold in three layers and tapped with a tapping rod for 35 strokes per layer. The strokes were done in such a way as to distribute the concrete evenly within the mold. After the top layer, the concrete was brought to the finish level using the trowel or knife edge. The concrete was allowed to set for 24 hours. After 24 hours, the cubes were released from the metal molds and cured by putting them in a curing tank filled with potable water above the cubes and allowed to be in the tank for a period of 27 days. After 27 days, the cubes were brought out of the tank and the surface water on them was allowed to dry; they were weighed to obtain their individual weight and then crushed to obtain the crushing load of each, which in turn will be used to determine their respective compressive strength. The crushing was done in a compression testing machine. A cube was placed on the machine and loads were applied gradually until the concrete cube failed. The load at which it failed is the crushing load. The procedure for the crushing was repeated for 14-day and 28-day cubes.

Calculations

The compressive strength is calculated by dividing the crushing loads by the cross-sectional area of the concrete cube.

Modeling and Optimization

Regression Analysis

$$Y_j = \beta_0 + \beta_j X_j + \epsilon_j \quad (3.1)$$

for $j = 1, 2, \dots, n$

where

n is the number of observations on both dependent and independent variables

Y_j is the observation of y (Slump or compressive strength)

X_j is the j^{th} observation of the independent variables),

ϵ is an independent and normally distributed random error term with mean 0 and variance σ^2

β_0 is the intercept

β_j is the slope

Assumptions of Regression

1. The mean of ϵ in any particular period is zero. i.e. $E(\epsilon_j) = 0$
2. The variance of ϵ is constant for all values of x . i.e. $\text{var}(\epsilon_j) = \sigma^2 \forall X_j$
3. The random terms (ϵ_i, ϵ_j) are independent, i.e. $\text{cov}(\epsilon_i, \epsilon_j) = 0$
4. The random error is normally distributed, i.e. $E(\epsilon) \sim N(0, \sigma^2)$

Experimental Investigation

Selection of proportion and constraints

The IV-optimal design criteria are used to construct a design matrix to investigate the influence of four concrete components (cement (A), water (B), coarse aggregate (C), fine aggregate (D) on slump,

7, 14 and 28-day compressive strength (f'_c) of the concrete. The coarse-to-fine aggregate ratio range is between 1.5 and 2. The four components and their volume and mass fraction ranges are given in Table 4.10

Experimental design details

In the statistical mixture methodology, the measured responses are assumed to depend on the proportion of materials present in the mixture rather than on the amount of mixture. In general, in a mixture with q components where x_i represents the proportion of the i th ingredient in the mixture, the relation between variables is as in equation [1]. Therefore, the constraint in equation [1] renders the levels of factor x_i dependent, which makes the mixture experiment method different from the usual response surface or factorial experiments.

$$[1] \quad \sum_{i=1}^q x_i = x_1 + x_2 + x_3 + \dots + x_q = 1 \quad x_i \geq 0, i = 1, 2, 3, \dots, q$$

The experimental region for a mixture of q components is a simplex with q vertices in $q-1$ dimensions (degree). Theoretical, physical, or economical considerations often impose additional constraints, as in equation [2] on the individual components. The upper and lower bounds on the component properties are as follows:

$$[2] \quad 0 \leq L_i \leq x_i \leq U_i \leq 1 \quad i=1, 2, 3, \dots, q$$

Where L_i and U_i denote lower and upper bounds, respectively. In cases where ≥ 4 , lower and upper bounds make the experimental region more like an irregular polyhedron. As such, a computer-based algorithm is required to construct a design for such a region. Where prediction is important, the computer based IV-optimal design is recommended for generating experimental design points; it provides lower average prediction variance across the region of experimentation. The algorithm of IV-optimal design picks points that minimize the integral of the prediction variance across the design space (Myers and Montgomery, 2008; Smith, 2005); in this research, Wolfram regression software was used. According to Myers and Montgomery (2008), the properties of a good design can be grouped into a design and an analysis stage. Some properties can be integrated at the design stage (before any data is collected), but others cannot be checked and possibly adjusted after data collection and analysis. In the design stage, an appropriate experiment design depends on several criteria, such as generating a satisfactory distribution of information, being cost-effective, building an appropriate model, providing an estimate of repeatability, and being able to check the adequacy of the fitted model. Choosing a proper model that will adequately explain the data and explore the relationship between variables can lead the experimenter to achieve the “best” experimental design. The second-order Scheffé canonical polynomial is considered to have priority over the first-order model, as the literature indicates that blending effects (interactions) are mostly significant. Also, where optimization is considered to be important, it is better to use a second-order and higher-order model. The general form of a quadratic Scheffé polynomial is written as:

$$[3] \quad E(Y) = \sum_{i=1}^q \beta_i x_i + \sum_{i=1}^{q-1} \sum_{j=i+1}^q \beta_{ij} x_i x_j$$

Where $x_i x_j$ is referred to as the quadratic blending terms in the mixture experiments and coefficients β_{ij} are referred to as quadratic or nonlinear blending coefficients. Where $\beta_{ij} \neq 0$, it means blending between components (x_i and x_j) is synergistic (Smith, 2005). The number of terms in this model is the same as the number of components in the mixture, and the number of coefficients in the model is the basis for the number of mixtures to be designed.

To simplify the calculation and analysis, the actual variable ranges are transformed to dimensionless coded variables with ranges of ± 1 . The variables x_1 to x_4 are codified using the following formula

$$[4] \quad U_{\text{Pseudocomponent}} = (U_i - x_i)/(U - 1)$$

Where U_i is the upper bound for the i th component, X_i is the uncoded value, and U is the sum of upper bounds. When using this transformation, it should be kept in mind that the pseudocomponents have effects that are opposite those of the real components (Smith, 2005).

In this study, the aim was to develop regression models predicting the properties of interest as well as investigating the effect of four components and their blending effects to reach an optimal mix proportion. Commercially available regression software was used to select 20 mixtures as shown in Table 2. The software used the IV-optimal algorithm to search for the best available combinations of points satisfying the design region constraints that will yield the best prediction of responses in the design space, and for developing a quadratic model. In the mixture design method, the relative proportion of the components is usually considered to be by volume fraction rather than weight. Volume fractions can then be converted to their corresponding weight using the specific gravity obtained from laboratory testing or from the material suppliers. The mixtures were then tested in random order to obtain response data.

Materials

The cement used was blended cement produced by Dangote Cement, grade R42.5 3X, with a maximum nominal size of 19mm coarse aggregate. The fine aggregates were natural sand. The relative density value of the coarse aggregate is 2.88. Their absorption rate was 10.81%. The size gradings of coarse and fine aggregates are given in Table 4.12.

Mixing procedure, specimen fabrication and testing procedure

All mixes in Table 4.11, each approximately 0.01 m^3 in volume, were prepared in a laboratory pan mixer with a nominal capacity of 0.10 m^3 . The following procedure was used in the preparation of all laboratory mixtures. Cement and Fine aggregates are first mixed for 60 seconds, and coarse aggregate is added; then initial water is added and the whole component mixed. Initial mixing took place for 3 minutes. The mixing is then stopped for 3 minutes for absorption to bring the aggregates to a state of saturation. Mixing is then resumed for another 3 minutes, making sure a homogeneous mixture is obtained. Slump was measured immediately after mixing, following the ASTM C143 / C143M - 12 procedures. To obtain adequate consolidation, the cubes were filled in accordance with ASTM C31 / C31M - 12. The cubes were covered with plastic sheets and were left in the casting room at 20°C for 24 hours. Then they were stripped and kept inside the curing tank and a temperature of about $23 \pm 2^\circ\text{C}$ until testing. The gain in compressive strength was determined by testing $150\text{mm} \times 150\text{mm} \times 150\text{mm}$ ($6'' \times 6'' \times 6''$) cubes at 7, 14 and 28 days. Three cubes were tested at each age for each concrete mixture in accordance with ASTM C39 / C39M.

The summary of results for the final regression models is given in Tables 4.14 and 4.15 for slump, and 4.17 and 4.18 for 28-day compressive strength. These tables include mean square, model p-value, lack-of-fit p-value, R-squared (R^2), predicted R-squared (R^2_{pred}), adjusted R-squared (R^2_{Adj}), and prediction error sum of squares (PRESS). The p-values of the models, which are measures of how likely a null hypothesis can be rejected, are less than 0.005. In addition, the p-value of lack-of-fit is higher than the desired level of significance (0.005).

The models are all considered adequate in view of the acceptable p-values and reasonably high set of R^2 values. Linear models are fit to all responses except the slump and a natural logarithm transform was applied to model the slump. The resulting regression models are shown in Table 4.15.

Trace Plot

Trace plots have been widely used in mixture design methods to assess the effects of mixture components on the measured responses (Myers and Montgomery, 2008). In general, a trace plot can be drawn in the Cox direction as introduced by Cox (1971), which is an imaginary line projected from the reference mixture (usually the centroid) to the vertex $x_i=1$ (Smith, 2005). It reveals how the response changes with the variation of each component from its low to high setting in the design region, while keeping all others in the same relative ratio at a specified reference mixture, here the centroid. The horizontal and near-horizontal trace for a component in a trace plot usually suggests that this component does not affect the results. Conversely, the effect of a component with a trace that is clearly not horizontal could be significant. The above interpretations all rely on the variance of the effect.

Figure 4.2 illustrates the trace plot for slump, and fig 4.3 for 28-day compressive strength. As expected, water content has a positive effect on the slump. Surprisingly, an increase in cement content appears to reduce the slump. However, this apparent reduction may not be significant when compared to the effect of water and experimental error. Extremely fine particles in this type of blended cement may slightly reduce the workability of the mixtures. The coarse and fine aggregates have a negligible effect on variation of the slump. As expected, cement content has a positive effect on the compressive strength, while water content has a negative effect.

Numerical Optimization (Desirability Optimization Methodology)

One approach to optimize multiple responses is to use the techniques popularized by Derringer and Suich (1980). This numerical approach makes use of desirability functions. The general approach involves the conversion of each response (y_i) into an individual desirability function that varies over the interval $d_i = 1$. If the response (y_i) is in its acceptable range, then $d_i=1$, and if the response is outside of it, $d_i=0$.

Overall desirability, D , is defined as a geometric mean of the individual desirability function d_i over the feasible region of the mixture to measure the satisfaction of combined goals for all responses.

$$[5] \quad D = (d_1 d_2 d_3 \dots d_n)^{1/n}$$

Where “ n ” is the number of responses in the mixture. Depending on the objective for the responses, the individual desirability functions can be defined as “maximum, minimum, target, in range and equal to”. Also, the limitation on the lower and upper level of each component can be set.

For simultaneous optimization, it is possible to place more emphasis on the upper and lower bounds or to emphasize the target value by selecting additional parameters called weights that can be altered from 0.1 to 10. When the weight is equal to 1, the desirability function is linear. Choosing a weight greater than 1 places more emphasis on the goal and a weight less than 1 makes the goal less important. Furthermore, in the desirability objective function D , each response can be assigned an importance value relative to the other responses. The importance (r_i) varies from the least importance (+) a value of one, to the most importance (+++++) a value of 5. If varying degrees of importance are assigned to the responses, the overall desirability, D , is as follows:

$$[6] \quad D = (d_1^{r_1} \times d_2^{r_2} \times \dots \times d_n^{r_n})^{1/\sum r_i} = (\prod_{i=1}^n d_i^{r_i})^{1/\sum r_i}$$

Where n is the number of responses in the mixture. The numerical optimization finds a point that maximizes the desirability function in either equation [5] or [6] based on the goal and constraints on the responses.

In this study, the desired responses are defined as target and maximum by assigning a particular importance value. The mixture that was found to maximize desirability ($D = 0.82$), has the following proportion expressed in mass fraction: cement = 420 kg/m³, water 164 kg/m³, coarse aggregate = 1126 kg/m³, fine aggregates = 671 kg/m³. The predicted response values for the optimum mixture are given in Table 4.15b. The predicted mixture was prepared after 3 months with the same materials and under almost the same testing conditions to verify the performance of the statistical mixture methodology. Confirmation tests and 95% prediction intervals are also given in Table 4.15. The experimental verification results all fall inside the prediction intervals and show that the constructed models work effectively.

Presentation and Analysis of Results

Table 4.12: Mix Proportions for Experimental Mixture (per cubic meter of concrete)

Design ID	Run Order	Cement (kg/m ³)	Water (kg/m ³)	Coarse Agg. (kg/m ³)	Fine Agg. (kg/m ³)	w/c ¹	CA/FA ²
6	1	401	164	1088	725	0.41	1.50
5	2	444	155	1127	670	0.35	1.68
15	3	444	155	1099	701	0.35	1.57
0	4	411	161	1104	711	0.39	1.55
4	5	415	164	1127	674	0.39	1.67
7	6	438	162	1099	686	0.37	1.60
16	7	444	155	1114	686	0.35	1.62
0	8	411	161	1104	711	0.39	1.55
12	9	391	164	1118	708	0.42	1.58
7	10	438	162	1099	686	0.37	1.60
11	11	426	159	1127	680	0.37	1.66
10	12	444	164	1070	705	0.37	1.52
9	13	371	162	1111	736	0.44	1.51
3	14	441	158	1078	713	0.36	1.51
2	15	382	158	1127	719	0.41	1.57
8	16	412	155	1101	725	0.38	1.52
17	17	444	164	1108	725	0.37	1.53
14	18	416	160	1088	725	0.38	1.50
13	19	378	156	1127	725	0.41	1.55
1	20	412	164	1112	725	0.40	1.53

w/c: water/cement ratio

CA/FA: coarse to fine aggregate ratio

Table 4.13 Fractional Presentation of the Mix Proportion for Test Results Experimental Mixtures

Cement	Water	Coarse Agg	Fine Agg	Y1 Slump	Y2 _a =7ays Compressive Strength	Y2 _b =28ays Compressive Strength	Y2 _c = 28ays Compressive Strength
0.17	0.07	0.46	0.3	145	24.39	32.03	43.16
0.19	0.06	0.47	0.28	75	30.18	37.90	50.47
0.19	0.06	0.46	0.29	24	28.91	36.47	46.94
0.17	0.07	0.46	0.3	54	24.64	31.70	43.85
0.17	0.07	0.47	0.29	140	25.71	31.81	45.2
0.18	0.07	0.46	0.29	72	27.69	34.25	48.92
0.19	0.06	0.46	0.29	30	28.75	31.49	45.73
0.17	0.07	0.46	0.3	48	26.66	33.28	43.93
0.16	0.07	0.47	0.3	87	23.96	30.92	41.78
0.18	0.07	0.46	0.29	140	27.57	32.66	46.71

0.18	0.07	0.47	0.28	50	27.43	36.27	46.76
0.19	0.07	0.45	0.3	73	26.70	34.28	47.12
0.16	0.07	0.47	0.3	150	20.30	26.92	38.85
0.18	0.07	0.45	0.3	97	28.45	36.44	48.74
0.16	0.07	0.46	0.31	100	24.96	30.08	44.12
0.17	0.06	0.46	0.31	70	27.36	35.59	47.05
0.18	0.07	0.45	0.3	23	29.36	38.53	46.45
0.17	0.07	0.46	0.3	27	26.96	35.88	47
0.16	0.07	0.47	0.3	75	24.14	29.30	45.44
0.19	0.07	0.46	0.3	135	26.15	33.84	46.32

Regression Analysis: Y1_Slump versus Cement, Water, Coarse Agg, Fine Agg

The regression equation is

$$Y1_Slump = -4010 + 2414 * Cement + 5750 * Water + 5018 * Coarse_Agg + 3238 * Fine_Agg$$

.....(4.1)

Table 4.14: Individual Coefficient contribution for estimating Slump

Predictor	Coefficient	SE	T value	P-value
	Coefficient			
Constant	-4010	2232	-1.8	0.093
Cement	2414	1937	1.25	0.232
Water	5750	2815	2.04	0.059
Coarse Agg	5018	2679	1.87	0.081
Fine Agg	3238	2283	1.42	0.177

Standard Error = 39.7668 R-Sq = 61.5% R-Sq(adj) = 43.2%

The result obtained in Table 4.14 shows a positive coefficient of determination (R-square =61.5%), which implies that the independent variables were able to explain about 61.5% of the total variation in slump. Also, the coefficients for all the independent variables were positive, which indicates that the variables have a positive relationship with the response variable, Slump. The appropriate model for estimating slump was found to be equation (4.1).

Table 4.15: Analysis of Variance (ANOVA) for estimating Slump

Source of variation	Degree of Freedom	Sum of Squares	Mean Square	F-value	P-value
Regression	4	10913	2728	1.73	0.197
Residual Error	15	23721	1581		
Total	19	34634			

The result of the ANOVA analysis presented in Table 4.14 found an F-value of 1.73 and a p-value of 0.197. This result indicates that the independent variables do not significantly impact the model since the p-value =0.197 is greater than the significance level of 0.05, assuming a 95% confidence level.

Table 4.16: Table showing distribution of observed slump, Predicted Slump and Residual by run order

Run Order	Y1 Slump	RESIDUAL	Y1 Slump (PREDICTED)
1	145	62.22197	82.77803
2	75	16.01746	58.98254
3	24	-17.1807	41.18074
4	54	-28.778	82.77803

5	140	39.42017	100.5798
6	72	-2.54041	74.54041
7	30	-11.1807	41.18074
8	48	-34.778	82.77803
9	87	-21.8174	108.8174
10	140	65.45959	74.54041
11	50	-42.3422	92.34221
12	73	-7.87968	80.87968
13	150	41.18256	108.8174
14	97	40.26139	56.73861
15	100	8.984355	91.01565
16	70	12.34402	57.65598
17	23	-33.7386	56.73861
18	27	-55.778	82.77803
19	75	-33.8174	108.8174
20	135	3.939839	131.0602

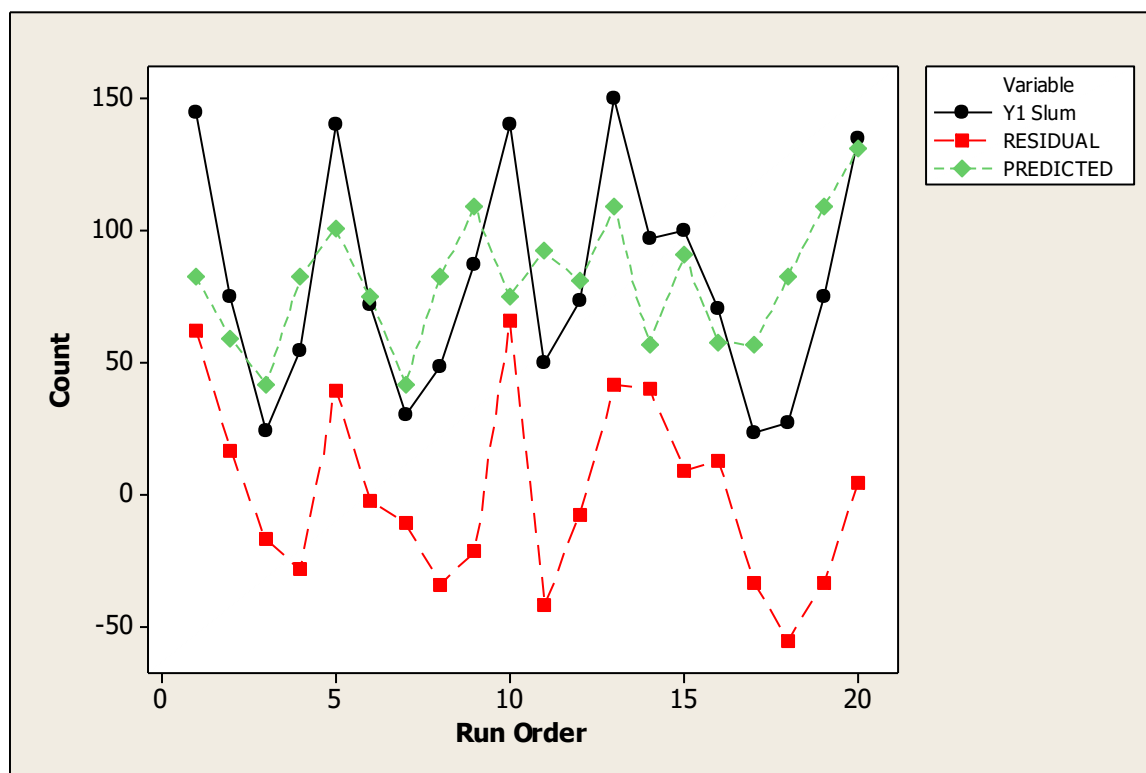


Figure 4.2: Graph showing distribution of observed slump, Predicted Slump and Residual by run order

The result obtained in Table 4.15 and Figure 4.2 revealed that the model is good for estimating slump since the predicted slump values have a related trend and minimum variation with regard to the run order.

Regression Analysis: Y2c_Compressive_Strength versus Cement, Water, Coarse Agg and Fine Agg

The regression equation is

$$Y2c_Compressive_Strength = 155 + 54.1 * Cement - 119 * Water - 158 * Coarse_Agg - 128 * Fine_Agg \dots \dots \dots (4.2)$$

Table 14.17: Individual Coefficient contribution for estimating Compressive Strength

Predictor	Coefficient	SE Coefficient	T value	P-value
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Constant	155	115	1.35	0.198
Cement	54.1	99.8	0.54	0.596
Water	-119	145	-0.82	0.425
Coarse Agg	-157.5	138	-1.14	0.272
Fine Agg	-128	117.6	-1.09	0.294

Standard Error = 2.04839 R-Sq = 72.8% R-Sq(adj) = 59.0%

The result obtained in Table 4.16 shows a strong positive coefficient of determination (R-square =72.8%), which implies that the independent variables were able to explain about 72.8% of the total variation in compressive strength. Also, cement was found to have a positive relationship with compressive strength, while variables such as water, coarse aggregate and fine aggregate were found to have an inverse relationship with compressive strength since their corresponding coefficients were found to be 54.1, -119, -157.5 and -128, respectively. The appropriate model for estimating compressive strength was found to be equation (4.2).

Table 4.18: Analysis of Variance for estimating compressive strength

Source of variation	Degree of Freedom	Sum of Squares	Mean Square	F-value	P-value
Regression	4	67.678	16.919	4.03	0.020
Residual Error	15	62.938	4.196		
Total	19	130.616			

The result of the ANOVA analysis presented in Table 4.17 found an F-value of 4.03 and a p-value of 0.02. This result indicates that the independent variables significantly impact the model since the p-value =0.02 is less than the significance value of 0.05, assuming a 95% confidence level.

Table 4.19: Table Showing Distribution of 28days Observed Compressive Strength, Predicted Compressive Strength and Residual by Run Order

Run Order	Y2c Compressive Strength	Residual	Predicted Compressive Strength
1	43.16	-1.819612326	44.97961233
2	50.47	2.232892644	48.23710736
3	46.94	-1.591898608	48.53189861
4	43.85	-1.129612326	44.97961233
5	45.2	0.515178926	44.68482107
6	48.92	2.118986083	46.80101392
7	45.73	-2.801898608	48.53189861
8	43.93	-1.049612326	44.97961233
9	41.78	-1.083419483	42.86341948
10	46.71	-0.091013917	46.80101392
11	46.76	0.253777336	46.50622266
12	47.12	-0.516799205	47.6367992
13	38.85	-4.013419483	42.86341948
14	48.74	1.644194831	47.09580517
15	44.12	0.961789264	43.15821074
16	47.05	2.160904573	44.88909543
17	46.45	-0.645805169	47.09580517
18	47	2.020387674	44.97961233
19	45.44	2.576580517	42.86341948
20	46.32	0.258399602	46.0616004

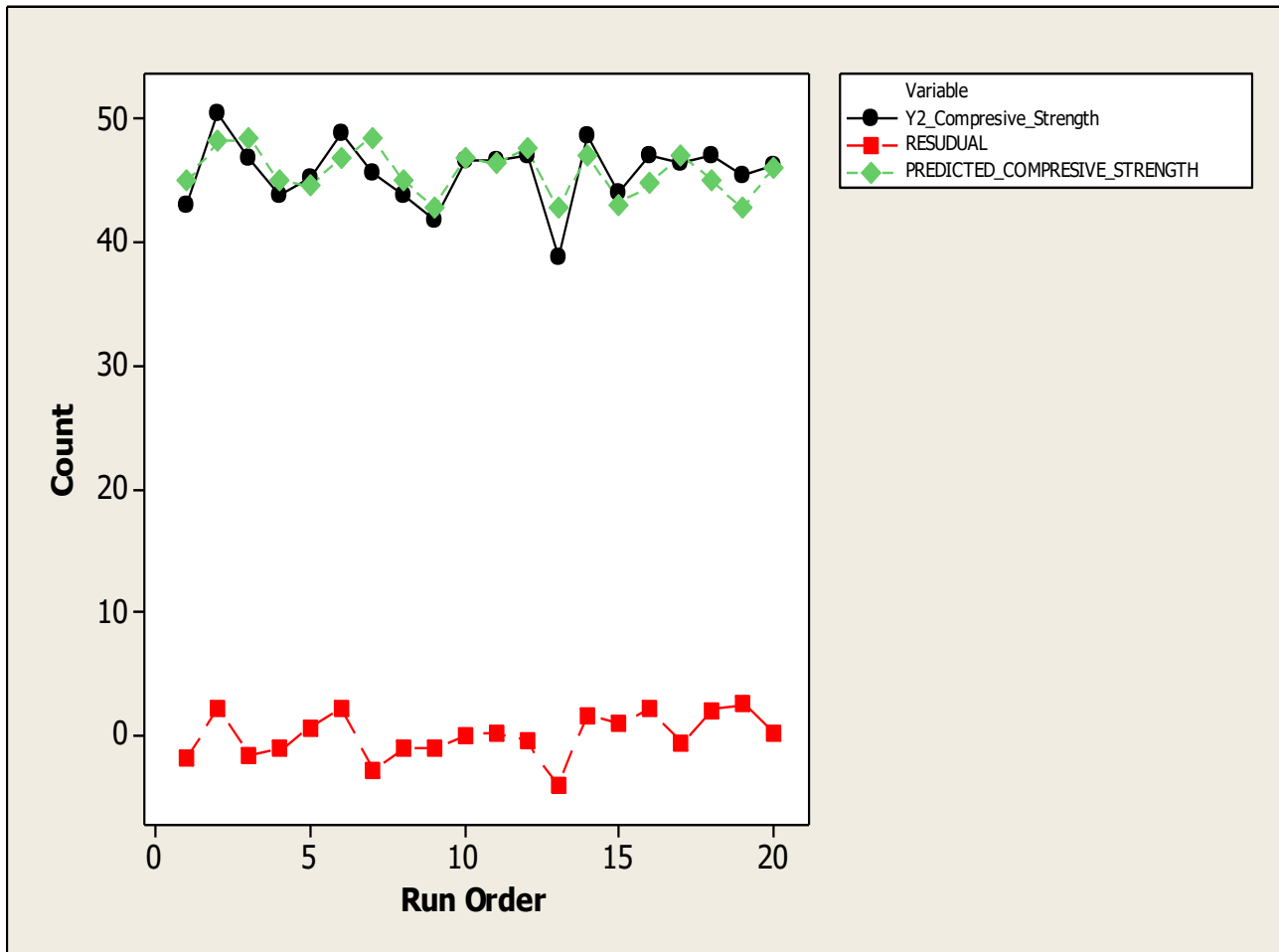


Figure 4.3: Graph showing distribution of observed compressive strength, predicted compressive strength and Residual by run order

The result obtained in Table 4.18 and Figure 4.3 revealed that the model is good for estimating compressive strength since the predicted compressive strength values have very minimal variation with regard to the run order.

Test for Normality of data using the Kolmogorov-Smirnov (KS) Test

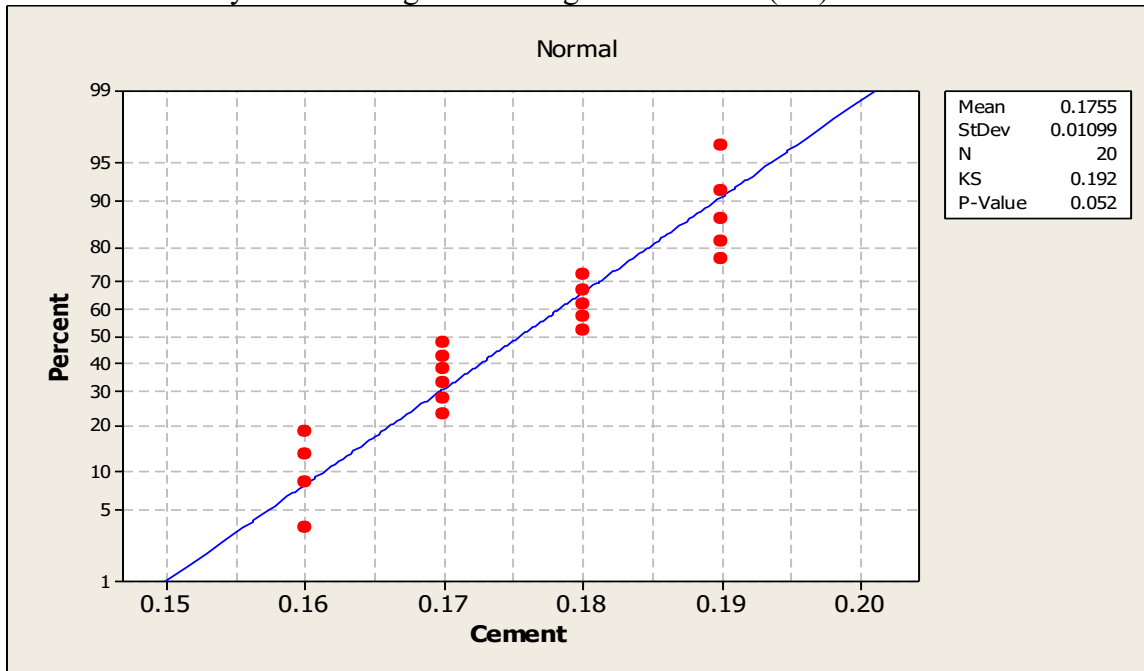


Figure 4.4: Normality test for Cement

The result found in Figure 4.4 shows a KS value of 0.192 and a p-value of 0.052, which implies that cement is normally distributed since the p-value = 0.052 is greater than the significance value of 0.05, assuming a 95% confidence level.

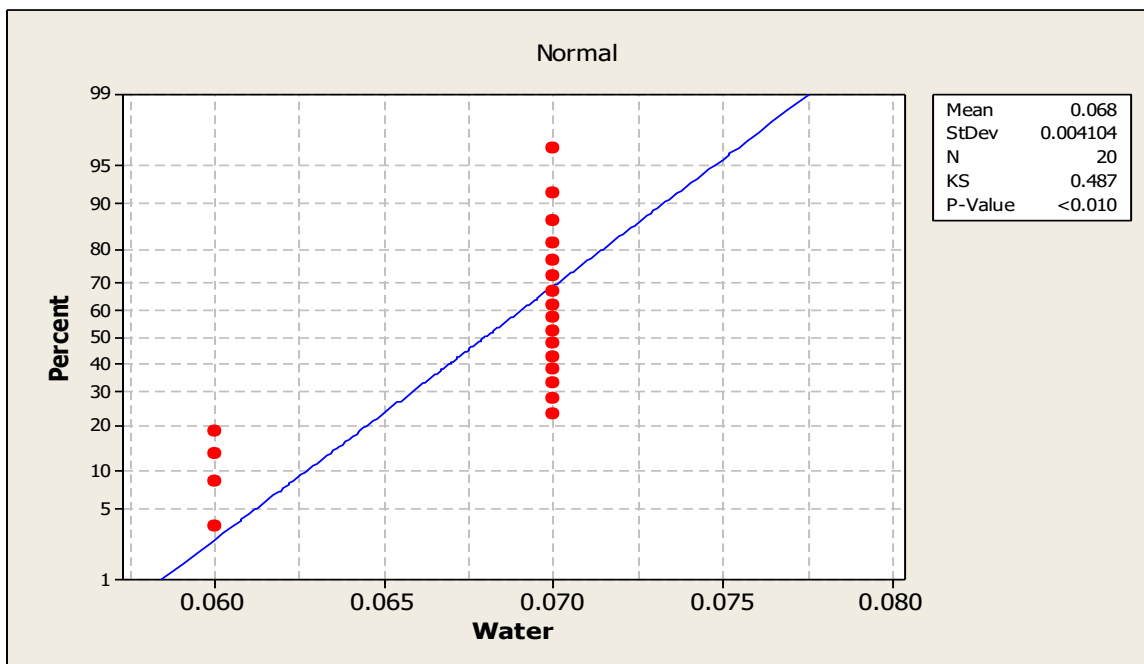


Figure 4.5: Normality test for Water

The result found in Figure 4.5 shows a KS value of 0.487 and a p-value of 0.01, which implies that water is not normally distributed since the p-value = 0.01 is less than the significance value of 0.05, assuming a 95% confidence level.

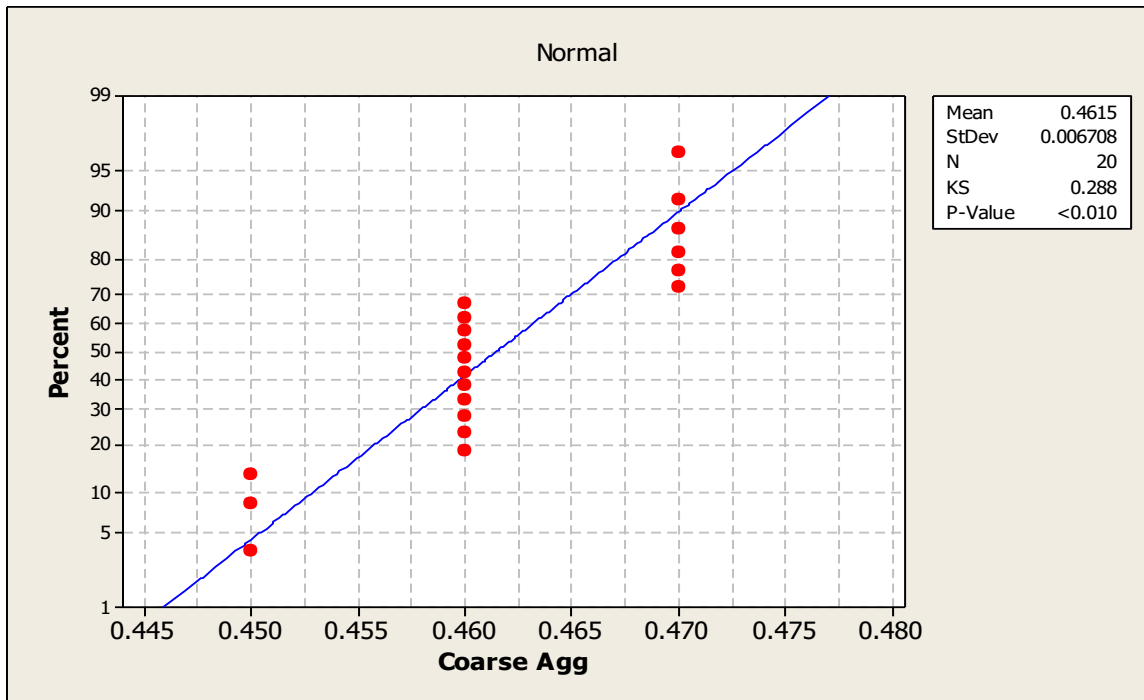


Figure 4.6: Normality test for Coarse Agg

The result found in Figure 4.6 shows a KS value of 0.288 and a p-value of 0.01, which implies that coarse Agg is not normally distributed since the p-value = 0.01 is less than the significance value of 0.05, assuming a 95% confidence level.

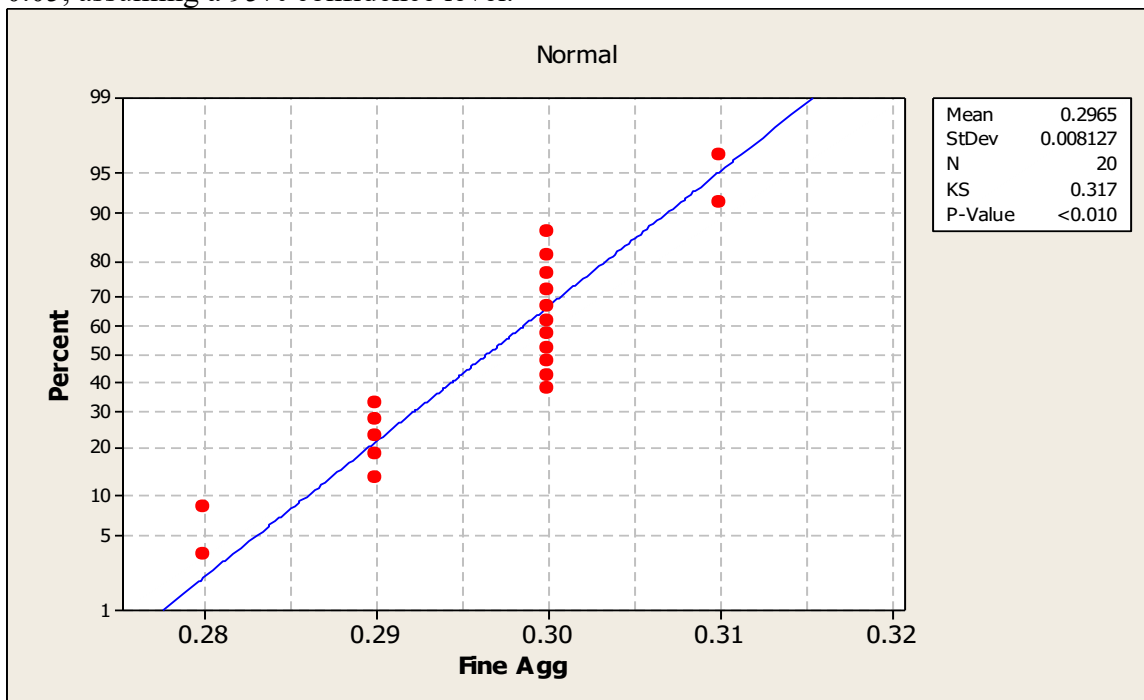


Figure 4.7: Normality test for Fine Agg

The result found in Figure 4.7 shows a KS value of 0.317 and a p-value of 0.01, which implies that fine agg. is not normally distributed since the p-value = 0.01 is less than the significance value of 0.05, assuming a 95% confidence level.

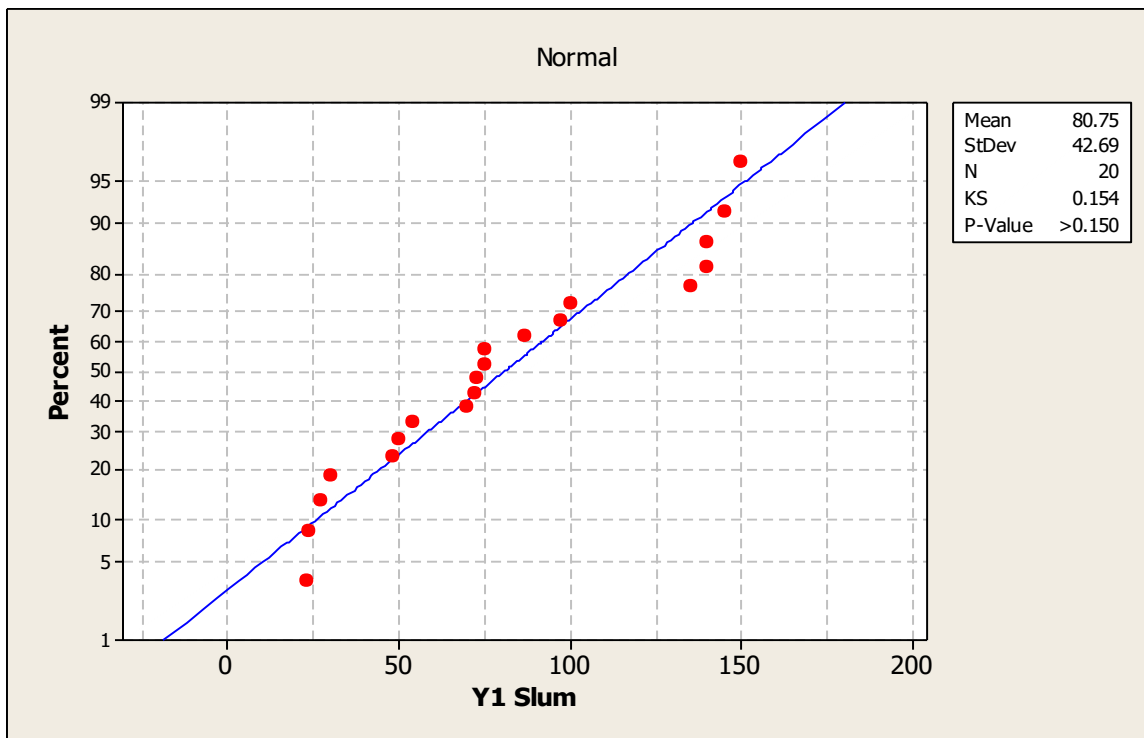


Figure 4.8: Normality test for Slump

The result found in Figure 4.8 shows a KS value of 0.154 and a p-value of 0.150, which implies that Slump is normally distributed since the p-value = 0.150 is greater than the significance value of 0.05, assuming a 95% confidence level.

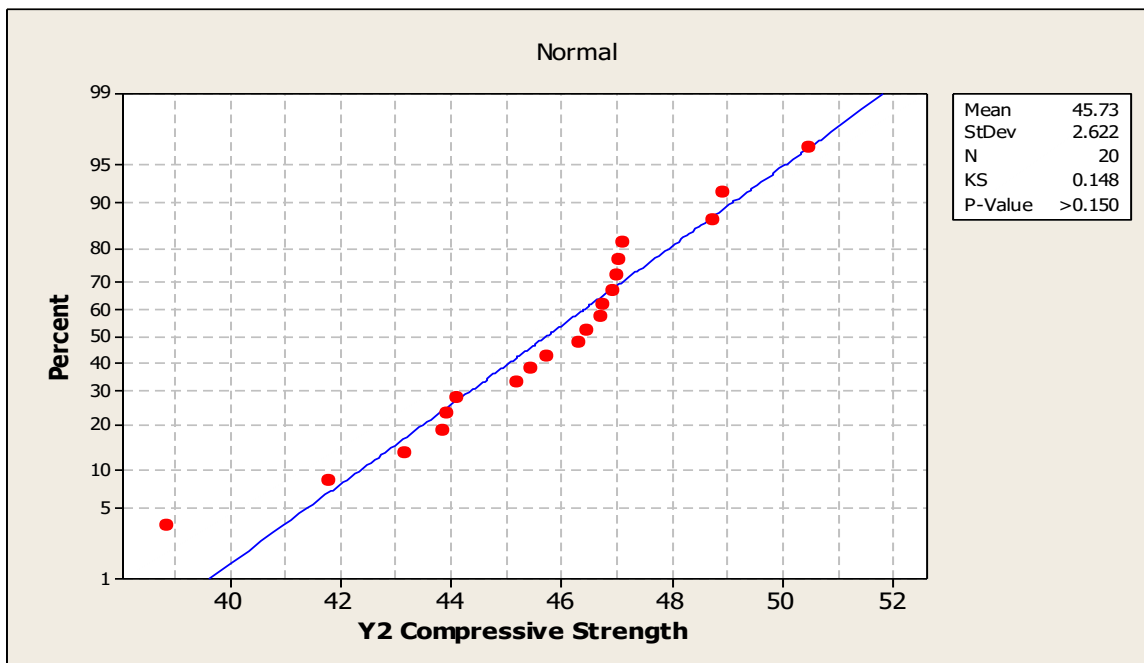


Figure 4.9: Normality test for compressive Strength

The result found in Figure 4.9 shows a KS value of 0.148 and a p-value of 0.150, which implies that compressive strength is normally distributed since the p-value = 0.150 is greater than the significance value of 0.05, assuming a 95% confidence level.

Independent variables such as Water, Coarse aggregate and Fine aggregate were found to be non-normally distributed. However, to satisfy the assumption of normality required before using the regression analysis, the variables will be transformed or standardized. Table 7 shows the standardized values of the variables.

Table 4.20: Table showing the standardized response of Cement, Water, Coarse Agg. and Fine Agg.

Run Order	SD Cement	SD Water	SD Coarse Agg	SD Fine Agg
1	0.170469	0.13012	0.467601	0.303614
2	0.179838	0.111341	0.471094	0.295509
3	0.161984	0.049474	0.462827	0.288912
4	0.16815	0.112352	0.463476	0.2959
5	0.1703	0.046855	0.460908	0.298145
6	0.175397	0.074458	0.457834	0.288064
7	0.168946	0.073787	0.463444	0.306477
8	0.181364	0.08223	0.468123	0.30229
9	0.179028	0.064452	0.460954	0.306318
10	0.18203	-0.00489	0.457959	0.29171
11	0.172564	0.112203	0.468411	0.294261
12	0.163461	0.012503	0.467754	0.292644
13	0.170187	0.05852	0.464438	0.29994
14	0.173487	0.152847	0.467805	0.298468
15	0.175061	0.094081	0.453431	0.294648
16	0.187147	0.089053	0.46746	0.296475
17	0.170662	0.105318	0.463688	0.283015
18	0.156982	0.04842	0.452441	0.29212
19	0.177343	0.046783	0.472231	0.302757
20	0.170854	0.049042	0.466499	0.294289

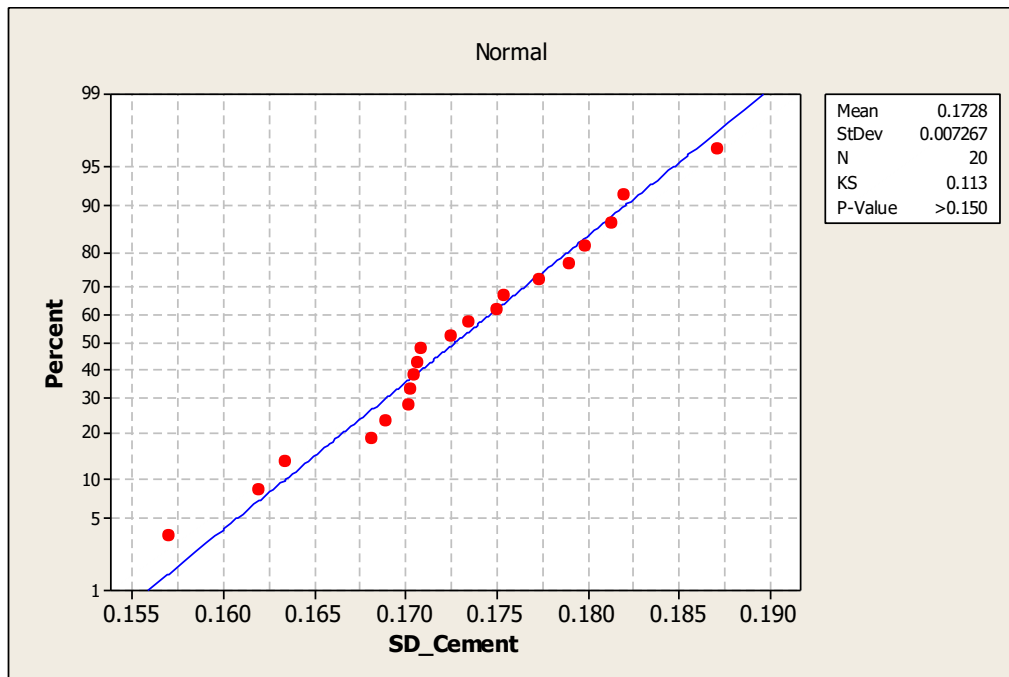


Figure 4.10: Normality test for standardized cement

The result found in Figure 4.10 shows a KS value of 0.113 and a p-value of 0.150, which implies that cement is normally distributed since the p-value = 0.150 is greater than the significance value of 0.05, assuming a 95% confidence level.

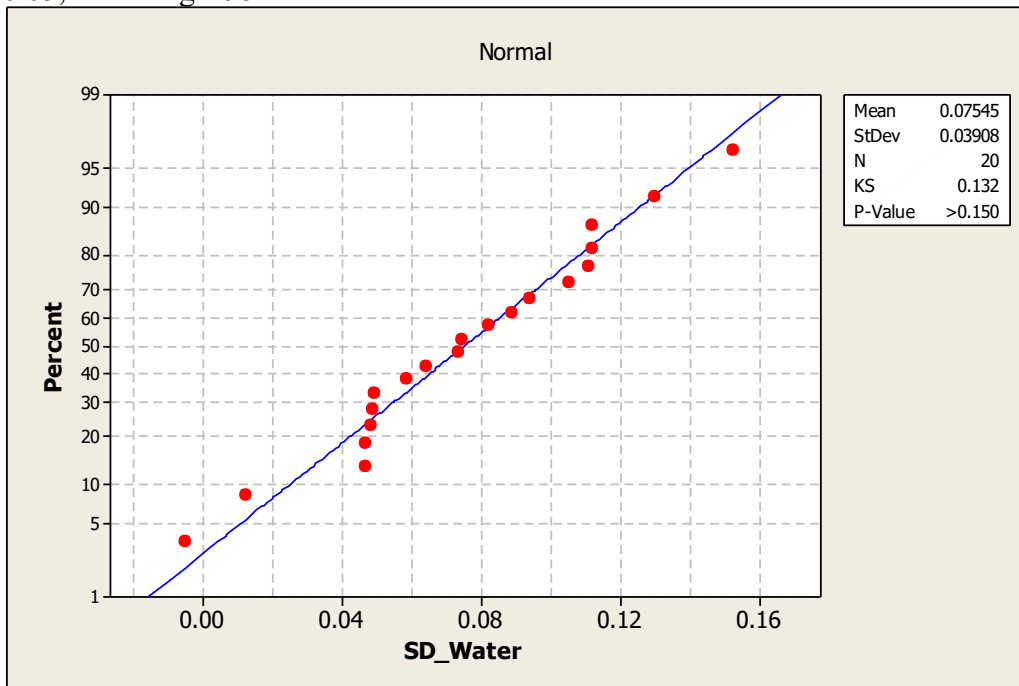


Figure 4.11: Normality test for standardized Water

The result found in Figure 4.11 shows a KS value of 0.132 and a p-value of 0.150, which implies that water is normally distributed since the p-value = 0.150 is greater than the significance value of 0.05, assuming a 95% confidence level.

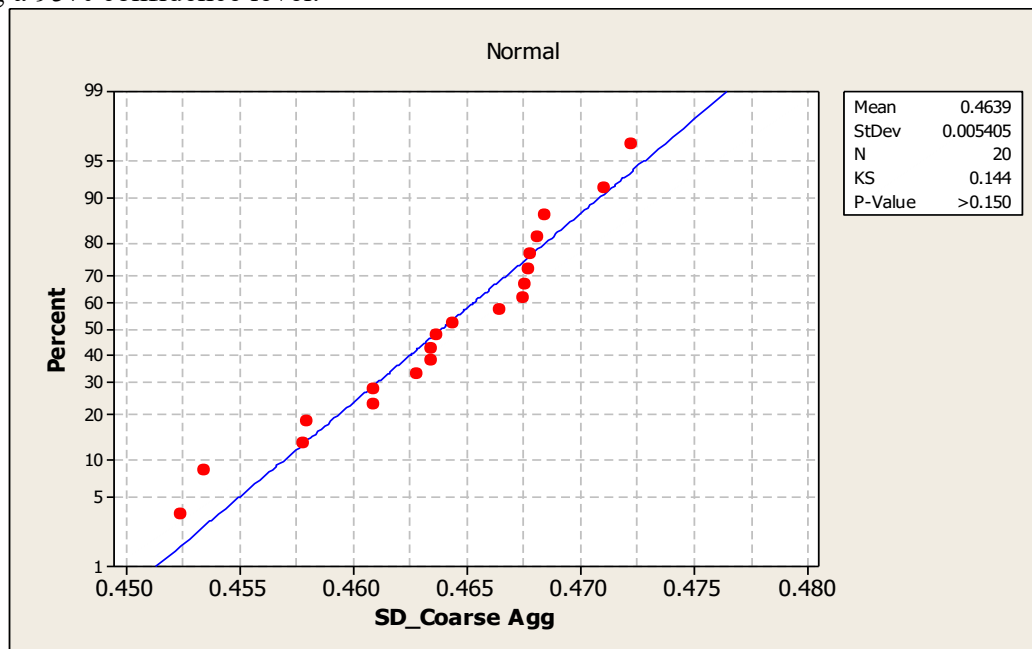


Figure 4.12: Normality test for standardized Coarse Agg.

The result found in Figure 4.12 shows a KS value of 0.144 and a p-value of 0.150, which implies that coarse agg. is normally distributed since the p-value = 0.150 is greater than the significance value of 0.05, assuming a 95% confidence level.

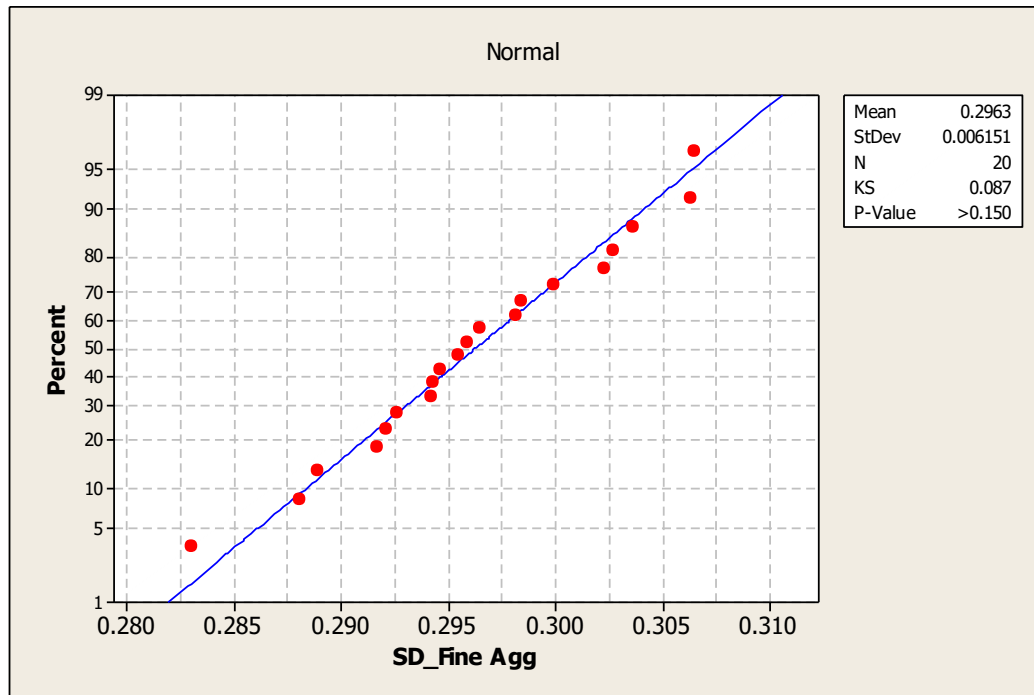


Figure 4.13: Normality test for standardized Fine Agg.

The result found in Figure 4.13 shows a KS value of 0.087 and a p-value of 0.150, which implies that fine agg. is normally distributed since the p-value = 0.150 is greater than the significance value of 0.05, assuming a 95% confidence level. Hence, the standardized variables can be used for the regression analysis.

4:2 Validation of Research

4:2:1: Optimization Results from Analyses and Validation Experiment

Table 4.14a Optimization Results from Analysis.

Item	Goal	Lower limit(L)	Upper limit (U)	Importance
Slump	85mm	41.18mm	108.82mm	3
28days f_c	48.53N/mm ²	42.86N/mm ²	48.53N/mm ²	4

Table 4.14b Prediction Results and Validation Experiment.

Item	Prediction values	Experimental values	95% Prediction Interval	
			Lower limit (L)	Upper limit (U)
Slump	85mm	94mm	41mm	131.06mm
28days f_c	48.53N/mm ²	47.73 N/mm ²	42.86N/mm ²	48.53N/mm ²

From Table 4.21b, it can be seen that the analysis from the ANOVA is satisfactory based on the results of the validation experiment, which was carried out using the regression analysis formulae.

References

American Concrete Institute. (1997). State-of-the-art report on high-strength concrete (ACI 363R-92, Reapproved 1997). American Concrete Institute.

- American Concrete Institute. (2009). Standard practice for selecting proportions for normal, heavyweight, and mass concrete (ACI 211.1-91, Reapproved 2009). American Concrete Institute.
- ASTM International. (2012). ASTM C31/C31M-12: Standard practice for making and curing concrete test specimens in the field. ASTM International.
- ASTM International. (2012). ASTM C39/C39M-12: Standard test method for compressive strength of cylindrical concrete specimens. ASTM International.
- ASTM International. (2012). ASTM C143/C143M-12: Standard test method for slump of hydraulic-cement concrete. ASTM International.
- Cox, D. R. (1971). A predictive model for polynomial regression. *Journal of the Royal Statistical Society: Series B (Methodological)*, 33(2), 218–253.
- Derringer, G., & Suich, R. (1980). Simultaneous optimization of several response variables. *Journal of Quality Technology*, 12(4), 214–219.
- Kanu, I. A. & Ndubisi, J. O. E. (2022). “Artificial intelligence and socio-economic development in Africa.” *OCHENDO: An African Journal of Innovative Studies*. 3(1), 2-21.
- Onyiah, S. C.; Ndubisi, J. O. E.; Kanu, A. I. (2024). “Navigating the Ethical Implications of Artificial Intelligence through Carol Gilligan’s Ethics of Care.” *Aquino Journal of Philosophy*. 4(2), 1-12.
- Kanu, A. I.; Addidi, T. D.; Kanu, C. C. (2024). “Artificial intelligence and cybercrime in Nigeria: Towards an Ethical framework”. *Dialogue and Universalism*, 32(1), 143-158.
- Kanu, I. A. & Okpokwasili, O. A. (2025). “Artificial Intelligence, Development and Ethics of Care: A Feminist Perspective on Human-Technology Relations”. Nwadiolor, K.; Ulpogu, N.; Adamy, J. N. (Ed.). *Artificial Intelligence, Ethics and Indigenous Wisdom: Rethinking Africa’s Cultural and Philosophical Future- Proceedings of the 2025 International Conference of the Association for the Promotion of African Studies (APAS)* (pp. 56-78). Abuja: Association for the Promotion of African Studies
- Mehta, P. Kumar, & Monteiro, Paulo J. M. (2014). *Concrete: Microstructure, properties, and materials* (4th ed.). McGraw-Hill Education.
- Montgomery, Douglas C. (2017). *Design and analysis of experiments* (9th ed.). John Wiley & Sons.
- Myers, Raymond H., Montgomery, D. C., & Anderson-Cook, C. M. (2009). *Response surface methodology: Process and product optimization using designed experiments* (3rd ed.). John Wiley & Sons.
- Yeh, I-Cheng. (1998). Modeling of strength of high-performance concrete using artificial neural networks. *Cement and Concrete Research*, 28(12), 1797–1808.
- Yeh, I-Cheng. (2007). Modeling slump flow of concrete using second-order regressions and artificial neural networks. *Cement and Concrete Composites*, 29(6), 474–480.